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A Monthly Based Bore Field Sizing Methodology with Applications to Optimum Borehole Spacing

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ABSTRACT

The required length of vertical ground heat exchangers (GHX) used in ground-coupled heat pump (GCHP) systems is determined so that the outlet temperature from the GHX remains within certain limits for the worst ground load conditions. These conditions may not necessarily occur after 10 or 20 years of operation, as is usually assumed, but often occur during the first year of operation.

The primary objective of this paper is to develop a general methodology for the calculation of the total required bore field length on a monthly basis during the first year of operation using the framework of the ASHRAE bore field sizing method. It is a three phase process. The first phase consists of analyzing and ordering ground loads according to the first month of operation. Next, a first set of required lengths is determined by using the analyzed ground load components and assuming a temperature penalty $T_p = 0$*. Then, an iterative process to calculate the temperature penalty at the end of each month is carried out to obtain the final required length for the worst conditions.*

The methodology is exemplified in a particular case with a slight annual cooling thermal imbalance and with a high influence of the hourly peak in heating. For this particular case, it is shown that the required bore field length occurs during the first year and that the starting month of operation has a strong influence on the results.

Finally, it is shown that it is possible to reduce the borehole spacing when the annual ground load is quasibalanced. In the case studied here, the minimum length occurs for a borehole-to-borehole spacing of about 3.2 m (10.50 ft).

INTRODUCTION

In ground-coupled heat pump (GCHP) systems, correct evaluation of the total ground heat exchanger (GHX) length and of the required borehole-to-borehole spacing is crucial to obtain good system performance. An undersized bore field may lead to operational problems because the return fluid temperature may be outside the operating temperature limits of the heat pump. Oversizing should also be avoided as it leads to unnecessary high capital costs.

A well-known procedure, based on the work of Kavanaugh and Rafferty (1997) and proposed by ASHRAE (ASHRAE 2007), is used by some software tools (e.g., presented in Philippe et al. [2010]) to determine the total bore field length. This procedure has been reformulated to take the following form (Bernier 2000).

$$L = \frac{q_h R_b + q_y R_{10y} + q_m R_{1m} + q_h R_{6h}}{\frac{(T_i + T_o)}{2} - (T_g + T_p)} \quad (1)$$

where L is the total bore field length ($L = nb \times H$ with nb as the number of boreholes and H the length of each borehole). The value of L can be regarded as the total bore field length required to avoid exceeding the minimum and maximum values of T_o, which is the inlet temperature to the heat pump (i.e., the outlet temperature from the GHX) for the worst ground load conditions.

Heat pumps can operate at temperature T_o as low as ≈ –5°C (23°F) in heating and as high as ≈ 40°C (104°F) in cooling. However, design engineers like to have a safety margin and will typically select values of T_o ≈ 0°C (32°F) in heating and ≈ 35°C (95°F) in cooling. The corresponding value of T_i, which is the

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outlet temperature from the heat pump (i.e., inlet temperature to the GHX) is typically 3 K to 5 K (5.4°F to 9°F) lower (in heating) or higher (in cooling).

Equation 1 is usually evaluated for both heating and cooling conditions yielding different bore field lengths. The larger of these two values is the required bore field length. However, in some cases, a smaller length is selected to reduce the capital cost of the GHX. This length is generally sufficient for one mode of operation (peak heating or cooling conditions) but supplemental devices (e.g., boiler or heat rejector) are required when operating at peak conditions for the other mode of operation. The GHX length of these hybrid systems can still be determined with Equation 1, however, the various terms need to be carefully evaluated, especially the duration of the peak condition.

The three consecutive ground load components are q_y, q_m, and q_h. The *ASHRAE Handbook—HVAC Applications* (2007) refers to these values as a series of constant heat-rate pulses. The maximum hourly ground peak load, q_h, is usually obtained from a knowledge of the building peak loads and of the heat pump coefficient of performance (COP) at the expected return fluid temperature from the GHX. Values of q_h in heating and in cooling need to be evaluated. The average monthly ground loads, q_m, one in heating and the other in cooling, are evaluated during the months when q_h occurs. The yearly average ground load q_y is the net result of heat rejection and collection into the ground. The same value of q_y is used in Equation 1 to determine the length in heating and in cooling. If the value of q_y is zero, then the ground loads are said to be balanced on an annual basis. Finally, peak hourly loads are the most important for GHX sizing because the first and fourth terms in Equation 1 typically represent 60% to 80% of the design length.

The terms R_{10y}, R_{1m}, and R_{6h} represent, respectively, the effective ground thermal resistances of the yearly, monthly, and hourly ground load components. As the subscripts suggest, these ground thermal resistances are evaluated for 10 years, 1 month, and 6 hours. These ground load step durations are recommended by ASHRAE (ASHRAE 2007), but it is also possible to use other step durations. The effective ground thermal resistances are evaluated using the cylindrical heat source (CHS) solution to heat transfer from a cylinder and the principle of superposition (Bernier 2000). Philippe et al. (2010) evaluated these three ground thermal resistances for a wide range of borehole radii and ground thermal diffusivities. They found that the range of variation of $R_{10y} \cdot k$, $R_{1m} \cdot k$, and $R_{6h} \cdot k$ are 0.379 to 0.382, 0.30 to 0.38, and 0.10 to 0.30, respectively, where k is the ground thermal conductivity in $W \cdot m^{-1} \cdot K^{-1}$ ($Btu \cdot h^{-1} \cdot ft^{-1} \cdot {}^\circ F^{-1}$) and the effective ground thermal resistances are given in $m \cdot K \cdot W^{-1}$ ($h \cdot ft \cdot {}^\circ F \cdot Btu^{-1}$). The borehole thermal resistance is given by R_b and is multiplied by the hourly peak load as shown in Equation 1. Typical values for R_b range from 0.05 to 0.2 $m \cdot K \cdot W^{-1}$ (0.09 to 0.35 $h \cdot ft \cdot {}^\circ F \cdot Btu^{-1}$).

The undisturbed ground temperature T_g is corrected for thermal interference between boreholes using a temperature penalty, T_p. Recognizing the deficiencies in the proposed calculation of T_p in *ASHRAE Handbook—HVAC Applications*, Bernier et al. (2008) suggested calculating T_p based on g-functions with the following equation:

$$T_p = \frac{q_y / L}{2\pi k_s} \cdot (g_n - g_1) \tag{2}$$

where g_n and g_1 are g-functions for the entire bore field and for one borehole, respectively. Both values depend on B, the borehole-to-borehole spacing; t, the time at which T_p is evaluated (typically 10 years span); and the bore field geometry. It should be noted that q_y impacts the bore field length in two ways. First, directly in the $q_y R_{10y}$ term, and secondly through T_p. The value of T_p can be relatively high, especially for small borehole-to-borehole spacings and large values of q_y. However, when ground loads are balanced on an annual basis, then $q_y = 0$, $T_p = 0$, and L (Equation 1) is independent of borehole-to-borehole spacing. However, it is not possible to neglect the influence of borehole-to-borehole spacing as monthly ground load imbalances will lead to borehole thermal interference and non-negligible values of temperature penalties. Unfortunately, Equation 1 cannot account for these situations because it does not account for monthly ground load imbalances.

It should also be noted that the denominator in Equation 1 is rarely the same for the determination of the heating and cooling lengths. For example, in a northern climate, $T_g \approx 10°C$ (50°F) and with typical values of T_m (=($T_i + T_o$) / 2) of 0°C and 35°C (32°F and 95°F) in heating and cooling, respectively, then the denominator is 2.5 times higher in cooling than in heating (assuming for this example that $T_p = 0$). Thus, the heating length might be the largest of the two even though the peak ground loads might be lower than for the cooling case. Furthermore, it should be noted that a perfectly balanced annual ground load ($q_y = 0$) does not lead to equal cooling and heating lengths.

A few aspects related to Equation 1 have not been investigated thoroughly and they are the subject of the present investigation. First, in certain cases, the maximum length is required during the first year of operation. This situation can occur when a system is started in winter and has not yet benefited from heat rejection into the ground. Equation 1 is not well-adapted for an evaluation of the required length during the first year and needs to be reorganized to account for monthly variations. Secondly, when the annual ground load is balanced or quasibalanced ($q_y \approx 0$), to what extent is it possible to reduce the borehole-to-borehole spacing (B) without increasing the required length? Finally, it should be noted that it is possible to obtain effective ground thermal resistances that account for axial effects and borehole thermal interference by using g-functions instead of the G-factor from the CHS solution (Ahmadfard and Bernier 2014). Considering that the temperature penalty is a good indicator of borehole thermal interference, it is sometimes preferable to use Equation 1 for a better interpretation of results and ease-of-use.

The main objective of this paper is to establish a general methodology, based on Equation 1, that would include an evaluation of the required length throughout the first year of operation at the end of each month. The methodology can be used regardless of the ground load profile for both balanced and imbalanced ground loads. Then, the methodology is applied to a quasibalanced ground load profile and the length is determined during the first, and tenth year of operation for two different starting months of operation. Finally, the methodology is used to study optimum borehole spacing to minimize total bore field length for a particular case.

LITERATURE REVIEW

Calculation of *L*

The first attempt to simplify the calculation of the required bore field length can be traced back to the pioneering work of Claesson and Eskilson (1988). In their methodology, the heat extraction is characterized by three components: a constant value with a superimposed yearly periodic component and a daily pulse. The borehole temperature is obtained based on analytical solutions giving the effective ground thermal resistances for each of these three components. The constant load and daily pulse terms are obtained from the asymptotes of the finite line source (FLS) analytical solution for the given time intervals. The ground thermal resistance associated with the periodic component results from the amplitude of its periodic line source solution. One year is normally considered for the periodic component, but no time period is suggested for the remaining components (Claesson and Eskilson 1988). An alternative to this method is also presented by Claesson and Eskilson in which the heat extraction is considered as a sequence of *n* stepwise constant values.

The ASHRAE method described previously also uses three load components (or *pulses*, according to the ASHRAE nomenclature) approach. However, the three effective ground thermal resistances are evaluated with the 1-D (radial) CHS solution. This is convenient because the effective ground thermal resistances do not depend on borehole length. Borehole thermal interference is accounted for by using a temperature penalty, T_p, which, in effect, corrects the undisturbed ground temperature. Bernier et al. (2008), Fossa (2011), and Capozza et al. (2012) have shown that the evaluation of T_p as suggested in *ASHRAE Handbook—HVAC Applications* (ASHRAE 2007) is deficient and proposed alternative calculation methods for T_p that improve the accuracy of Equation 1. Cullin et al. (2014, 2015) have compared the results obtained from the ASHRAE method and a simulation-based method with experimental data from four different systems, including a system that has been in operation for five years (Montagud et al. 2011). The *ASHRAE Handbook—HVAC Applications* equation yielded lengths with errors in the range from –21% to +103% while the simulation-based method yielded maximum errors of ±6%. Cullin et al. explain that the load representation in the ASHRAE method may explain the large differences. Only about 9% of the discrepancy could be due to inaccuracies in the input variables (measured data at the site). The results of this study suggests that a better load representation is needed in the ASHRAE method. The impact of the magnitude and the duration of the load on the bore field length was also investigated by Cullin and Spitler (2011).

The available design tools for borehole sizing, such as the Earth Energy Designer (EED) (Hellström and Sanner 1994) or Ground Loop Heat Exchanger Design Software (GLHEPRO) (Marshall and Spitler 1994; Spitler, 2000), are based on an iteration of the fluid temperature prediction where the total bore field length is adapted to fulfill the minimum and maximum entering fluid temperature constraints. These tools allow the designers to evaluate the required length at any particular moment during the operation of the system and also to select the starting month.

Some other authors performed simulations in TRNSYS (TRNSYS 2015) using the duct storage model (DST) to determine the borehole length for given temperature constraints. Shonder et al. (2000) used this technique to evaluate existing bore field design tools, including some that use the ASHRAE sizing equation. The results showed that three of the design tools agree within 12% of the length obtained with the DST model. Calculation of the required borehole length for the first and tenth year of operation were examined. In that particular case, the maximum length is for the heating mode and it occurs after 10 years of operation. The length predicted on the tenth year of operation is about 7% higher than the results from the first year of operation. The one-year design length is recommended when either the thermal conditions are roughly balanced or the multiyear effects are modest.

Sutton et al. (2002) developed the multilayer bore field design algorithm (MLBDA), which models radial heat transfer in a series of layers for cases where there is geological stratification in the ground. In this model, the borehole length is given as an initial value and then an iterative process is set to determine the length satisfying the entering fluid temperature constraints. The MLBDA algorithm was tested for both cooling and heating dominated systems using the data presented by Shonder et al. (2000). The borehole length was calculated after one, five, and 10 years of operation in a cooling dominated system in which the entering fluid temperature is varied from 29°C to 38°C (84.2°F to 100.4°F). The results showed that the maximum length occurred after 10 years of operation for this particular case. In terms of design lengths, the MLBDA method was within 3.0% and 7.4% of the reference case for cooling and heating, respectively.

Thermal Interference and Borehole Spacing

Claesson and Eskilson (1987) carried out a detailed study on the thermal influence between boreholes. They used a simplified approach based on an analytical solution to suggest rules of thumb to estimate borehole thermal interference. In their analysis, a single borehole is represented as an infinite line sink with a constant heat transfer rate per unit length, q_0. In a bore field, where boreholes are surrounded by a number of adjacent boreholes (N_{adj}) separated by a distance B, the thermal influence from the surrounding boreholes can be estimated by

$$\Delta T \cong \frac{q_0}{2\pi k_s} \cdot \frac{1}{2} N_{adj} \cdot E_1\left(\frac{B^2}{4\alpha t}\right) \qquad t > 0 \tag{3}$$

where $E_1\left(\frac{B^2}{4\alpha t}\right)$ represents the exponential integral.

Claesson and Eskilson applied Equation 3 to a reference example and concluded that the thermal influence is insignificant when the dimensionless factor $\frac{1}{2} N_{adj} \cdot E_1\left(\frac{B^2}{4\alpha t}\right)$ is lower than 0.1, while thermal influence should be taken as significant when this factor is higher than 1. No intermediate values between 0.1 and 1.0 are defined by Claesson and Eskilson (1987). Equation 3 has been applied to a few cases and the results are shown in Table 1.

As shown in Table 1, a reduction of the borehole spacing by a factor of 2 means that the time for the thermal influence to be significant is reduced by a factor of 4.

Equation 3 was initially formulated for a constant heat injection rate. However, in real situations, the ground load conditions vary in time and magnitude and this can be accounted for using superimposed variations. Claesson and Eskilson (1987) used an analytical solution with a balanced sinusoidal periodic line sink. As expected, the amplitude of the periodic temperature decreases as the distance from the borehole wall increases. The ratio between the amplitude at the borehole radius and at different distances from the borehole wall is evaluated for three time periods (one day, one month, one year). It is shown that for a borehole radius r_b = 0.055 m (2.16 in.) and a ground diffusivity α = 0.14 m^2/day (1.51 ft^2/day) the thermal influence of a daily periodic load is less than 1 m (3.28 ft), while a load with a periodic variation of one year has a small influence at distances greater than 5 m (16.4 ft). A completely negligible influence criterion, when the amplitude becomes (nearly) equal to its undisturbed value (Claesson et al. 1985), is defined as:

$$B > 2\sqrt{\alpha t_p} \tag{4}$$

However, this criterion was refined to include the contributions of the surrounding boreholes. The required distance B is set such that the amplitude of the periodic extraction/injection generated by the surrounding borehole is less than 10% of the amplitude caused by the borehole itself. For an infinite number of boreholes located in a rectangular grid the required borehole-to-borehole spacing to have an influence of less than 10% is given by:

$$B > 0.7\sqrt{\alpha t_p} \tag{5}$$

Table 2 shows the time period of the heat extraction for which there is negligible thermal influence for the two criteria established in Equation 4 and 5.

As shown in Table 2, the time period below which thermal influence is negligible increases by a factor of 8 between Equation 4 and 5.

Claesson and Eskilson (1987) pointed out that the criteria defined in Equation 3 are based on a two-dimensional solution, so the expression of the influence range is limited if the three-dimensional effects appear before the thermal influence criteria are met.

In Hellström's work, the periodic thermal process is defined by a periodic line source and the solutions are expressed as cylinder functions. The thermal influence between adjacent boreholes is estimated by comparing the temperature amplitude for a borehole in an infinite region and for a borehole in a circular region of radius r_1 in which a zero-heat flow boundary condition is set at the outer distance r_1 for the local thermal process. In Hellström's (1991) solutions, the lengths are written as dimensionless variables by scaling them with the penetration depth, as follows:

$$r' = \frac{r\sqrt{2}}{d_p} \tag{6a}$$

$$r'_b = \frac{r_b\sqrt{2}}{d_p} \tag{6b}$$

Table 1. Time Required to Reach a Significant Thermal Influence, According to Equation 3 (α = 0.14 m^2/day [1.51 ft^2/day])

Bore Field	$N_{adj,\,CB}$ for the Center Borehole	B, m (ft)	Time Required, days
3×1	2	3 (9.84) 6 (19.69)	64 257
3×3	8	3 (9.84) 6 (19.69)	16 64

Table 2. Time Period of Heat Extraction According to Equations 4 and 5

Criterion	B, m (ft)	t_p, days
$B > 2\sqrt{\alpha t_p}$	3 (9.8) 6 (19.7)	16 64
$B > 0.7\sqrt{\alpha t_p}$	3 (9.84) 6 (19.7)	132 525

$$r'_1 = \frac{r_1\sqrt{2}}{d_p} \qquad (6c)$$

The penetration depth for a periodic boundary temperature, $d_p = \sqrt{\frac{\alpha t_p}{\pi}}$, which was first introduced by Eskilson (1987), represents a characteristic length for the periodic thermal process in the ground.

In general, the thermal influence between adjacent boreholes becomes negligible when the penetration depth is small compared to the borehole spacing B (Hellström 1991). The criteria for negligible, moderate, and strong influence are defined for hexagonal and rectangular bore field patterns, as shown in Table 3.

With this analysis, it is possible to locate the boreholes closer to each other at a minimum distance with negligible influence between the boreholes and no impact on the design length.

Table 4 shows the period of the cyclic heat load that corresponds to different borehole spacing to have negligible, moderate or strong influence, according to the sorting criteria defined by Hellström (1991) for a value of $\alpha = 0.075$ m^2/day (0.81 ft^2/s). For a typical one-year periodic load, the influence becomes strong when the borehole spacing is smaller than 3.2 m (10.5 ft) and it is moderate for B smaller than 4 m (13.1 ft).

Table 3. Thermal Influence Limits for Hexagonal and Quadratic Pattern (Hellström 1991)

	Hexagonal Pattern	Rectangular Pattern
Negligible influence	$r'_1 = r_1\sqrt{\frac{2\pi}{\alpha t_p}} \geq 3$	$B \geq 2\sqrt{\alpha t_p}$
Moderate influence	$0.8 < r'_1 < 3$	$0.6\sqrt{\alpha t_p} < B < 2\sqrt{\alpha t_p}$
Strong influence	$r'_1 \leq 0.8$	$B \leq 0.6\sqrt{\alpha t_p}$

Table 4. Period Versus Strength of Influence for Different Borehole Spacings in a Rectangular Pattern (α = 0.075m^2/day [0.81 ft^2/s])

B, m (ft)	Negligible Influence	Moderate Influence	Strong Influence
6.0 (19.7)	$t_p \leq$ 4 months	4 months < t_p < 3.6 years	$t_p \geq$ 3.6 years
5.0 (16.4)	$t_p \leq$ 3 months	3 months < t_p < 2.5 years	$t_p \geq$ 2.5 years
4.0 (13.1)	$t_p \leq$ 2 months	2 months < t_p < 1.6 years	$t_p \geq$ 1.6 years
3.2 (10.5)	$t_p \leq$ 35 days	35 days < t_p < 1 year	$t_p \geq$ 1 year
3.0 (9.8)	$t_p \leq$ 30 days	30 days < t_p < 11 months	$t_p \geq$ 11 months

From the results for an hexagonal pattern, an optimum borehole spacing can be found from the analysis of the amplitude function of the temperature and the inverse amplitude of the periodic heat flow at the borehole radius for a series of values of $(r'_1 - r'_b)$ (Hellström 1991). The optimum corresponds to the maximum heat injection rate for a given temperature amplitude, or alternatively, it corresponds to the minimum amplitude temperature that can be obtained for a given heat injection rate.

Hellström related the value of r'_b and the dimensionless ratio of the surrounding circular region r'_1 for the minimum temperature amplitude in a hexagonal pattern. For instance, for $r_b = 0.05$ m (1.97 in.); $\alpha = 1.62 \cdot 10^{-6}$ m^2/s ($1.74 \cdot 10^{-5}$ ft^2/s); $t_p = 1$ year, r'_b is 0.018. The minimum amplitude results in approximately $r'_1 = 1.24$, resulting in $r_1 = 3.5$ m (11.5 ft). Therefore, the optimum borehole spacing is set to 7 m (23 ft) for this particular case of a hexagonal pattern. However, the estimation of an optimum borehole spacing for a rectangular pattern becomes more complex as the distances between boreholes are not equal (it is B or $\sqrt{2}\,B$).

Recently, some other authors have investigated the importance of an appropriate borehole spacing to control the annual ground thermal imbalance (Yang et al. 2013). Borehole spacings of 3, 4, 5, and 6 m (9.8, 13.1, 16.4, and 19.7 ft) were investigated. The center soil temperature varied from 19.5 to 23.8°C (67.1 to 74.8°F) when the borehole spacing was changed from 3 to 6 m (9.8 to 19.7 ft).

It may be advantageous to position the boreholes unequally within the bore field. This has been investigated by Cimmino and Bernier (2014b). The methodology uses the g-function to predict the impact of unequal borehole spacing on the total bore field length. Two bore fields of 3 × 7 and 5 × 10 boreholes are studied. The results show that moving the borehole toward or away from the centers has a negligible impact on the required bore field length because the g-function changes are insignificant.

METHODOLOGY

The objective of the proposed methodology is to obtain the required bore field length at the end of each month for the first year of operation and at the end of each month during the tenth year of operation. It is a three-phase process. First, ground loads need to be analyzed and ordered starting with the first month of operation. Then, using these loads and assuming a temperature penalty $T_p = 0$, a first set of required lengths is determined for each month. Finally, an iterative process is employed to calculate the temperature penalty in each month to obtain the final required length for the worst conditions.

Ground Load Analysis

As indicated previously in relation to Equation 1, design engineers often base their calculations on average monthly ground loads and peak ground loads. These are either the results of building hourly simulations combined with heat pump performance or simply based on peak monthly ground loads and an estimate of monthly ground loads. When hourly ground loads are available, it is convenient to reformat the loads so as to obtain an average and a peak ground load for each month. This can be done by simply averaging the ground loads over the month and searching for the peak load during that month. In the proposed methodology, it is assumed that a year is composed of twelve months, each with 30.42 days, for a total of 365 days.

Then, the designer has to decide on the duration of the peak load for each month. The impact of peak duration is not specifically studied in the present paper. However, it should be noted that the methodology presented as follows is adaptable to any peak duration. Once monthly loads (average and peak) have been determined, they need to be ordered starting with the month when the GCHP system is to be started.

Figure 1 illustrates an annual hourly ground load for a GCHP system located in a relatively cold climate but with significant cooling loads in the summer. This load profile will also be used in the application section later in this paper. Twelve horizontal bars in Figure 1 show the average monthly ground loads. The monthly peak heating and cooling ground loads are represented by circles and squares, respectively. It should be noted that some months, in the shoulder seasons, have both heating and cooling peaks. The maximum monthly average load is 105.37 kW (359.63 kBtu/h) in heating and –150.54 kW (–513.79 kBtu/h) in cooling. The maximum hourly peak loads in heating and cooling are 238.87 kW (815.26 kBtu/h) and –331.00 kW(–1129.70 kBtu/h), respectively. The annual average ground load is –7.71 kW (–26.31 kBtu/h), about 2.3% of the peak cooling load.

Determination of the Required Length for Each Month

The proposed methodology uses the same format as in Equation 1, but with different durations of the load components. The evaluation of the required length for month *i* takes the following form:

$$L_i = \frac{q_{h,i} \cdot R_b + \bar{q}_{pm,i} \cdot R_{pm,i} + q_{cm,i} \cdot R_{cm} + q_{h,i} \cdot R_h}{T_m - (T_g + T_{p,i})} \quad (7)$$

where L_i is the required bore field length, $q_{h,i}$ is the peak load for month *i*, $\bar{q}_{pm,i}$ is the average ground load for the preceding months (note that $\bar{q}_{pm,1} = 0$), $R_{pm,i}$ is the equivalent ground thermal resistance associated with the preceding months, $q_{cm,i}$ is the average load for the current month, $R_{cm,i}$ is the equivalent ground thermal resistance for the current month, $R_{h,i}$ is the equivalent ground thermal resistance for the peak load during month *i,* and $T_{p,i}$ is the temperature penalty at the end of month *i*. $T_{p,i}$ is based on the average ground loads of the previous months $\bar{q}_{pm,i}$.

Figure 2 illustrates the calculation process to evaluate the required length at the end of March (i.e., for *i* = 3) when the GCHP system starts to operate on January 1. For this example, the term $\bar{q}_{pm,3}$ corresponds to the average load of the previous two months, i.e., January and February. The terms $q_{cm,3}$ and $q_{h,3}$ are, respectively, the average monthly and peak loads for March. In this work, it is assumed that the peak load occurs at the end of the month, i.e., in the first few hours of the next month. This assumption, combined with the use of 30.42 days for each month, simplifies the design length equation because values of $R_{cm,i}$ are the same for every month. Thus, $R_{pm,i}$ is the only effective ground thermal resistance that changes from month to month.

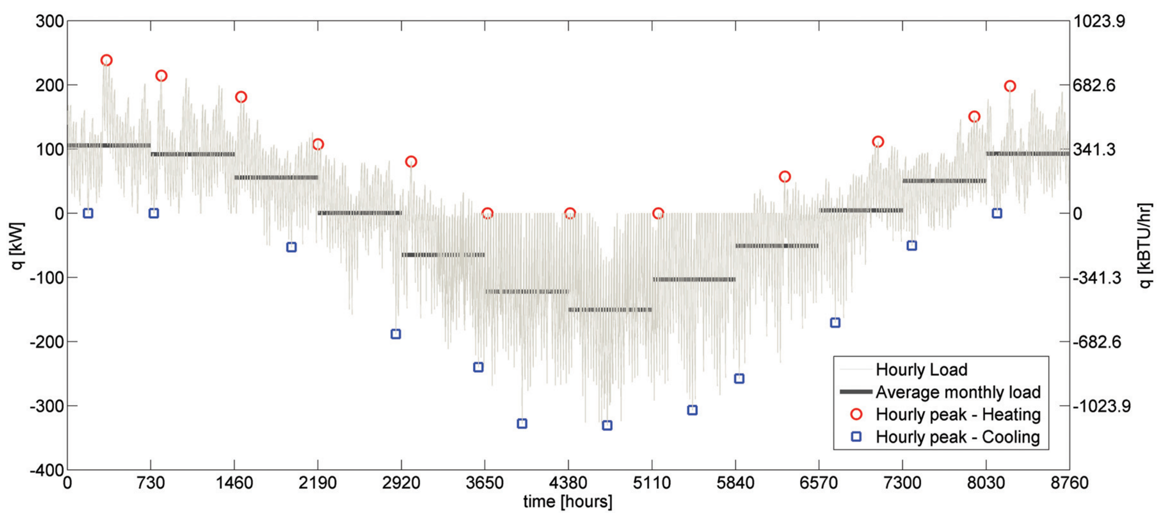

Figure 1 *Hourly ground heat load profile with a January starting month.*

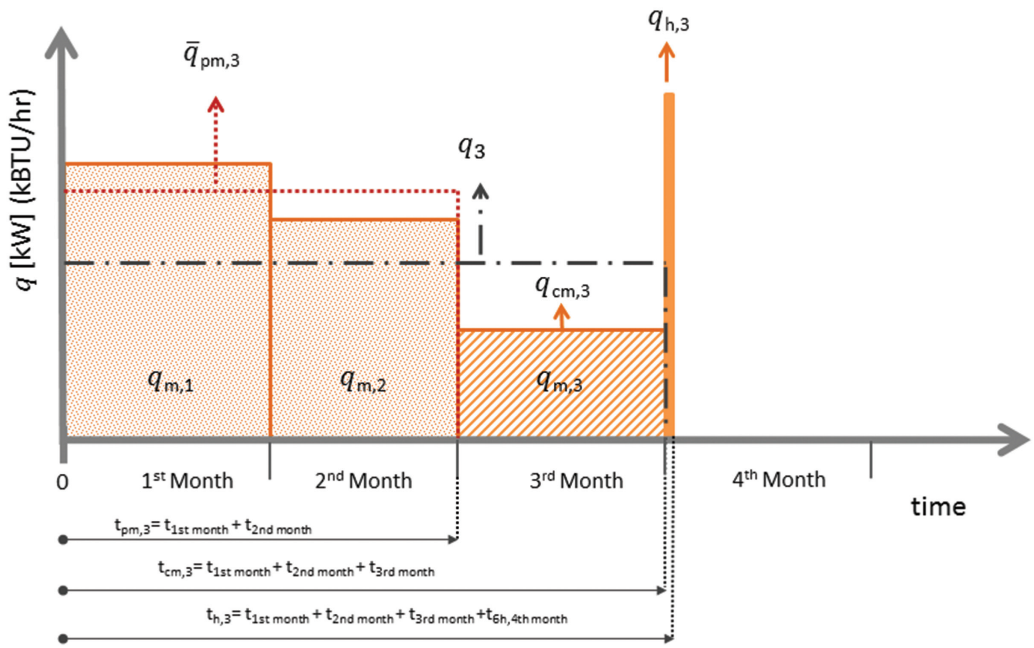

Figure 2 *Illustration of the calculation of various loads in Equation 7 for* i = 3.

The effective ground thermal resistances are calculated for their respective step duration. For this example ($i = 3$), the effective ground thermal resistances are written as

$$R_{pm,3} = \frac{G(\mathrm{Fo}_{t_{h,3}-0}) - G(\mathrm{Fo}_{t_{h,3}-t_{pm,3}})}{k_s} \tag{8a}$$

$$R_{cm,3} = \frac{G(\mathrm{Fo}_{t_{h,3}-t_{pm,3}}) - G(\mathrm{Fo}_{t_{h,3}-t_{cm,3}})}{k_s} \tag{8b}$$

$$R_{h,3} = \frac{G(\mathrm{Fo}_{t_{h,3}-t_{cm,3}})}{k_s} \tag{8c}$$

where Fo is the Fourier number, defined as $\mathrm{Fo} = 4\alpha t / d^2$, with the borehole diameter d as the characteristic length. Note that the various step durations included in Equations 8a–c are illustrated in Figure 2 for $i = 3$.

The G-factor is the analytical solution to transient heat transfer from an infinite cylinder in an homogeneous medium, often referred to as the CHS solution (Carslaw and Jaeger 1947). In this work, the G-factors are calculated using a correlation proposed by Bernier (2001). Also implicit in Equation 8 is the use of the principle of superposition (Bernier 2000). Finally, it should be noted that the CHS solution does not consider axial effects; only radial transient heat transfer is considered. According to Eskilson (1987) the radial approximation is valid with high accuracy for times less than $H^2 / 9\alpha$ which, for typical cases, corresponds to several years. Thus, the use of the G-factors for the first year of operation is justified.

As shown in the literature review, borehole thermal interference may be significant after a few weeks for small borehole spacings. In the proposed methodology (Equation 7), borehole thermal interference is evaluated at the end of each month using the concept of temperature penalty. Values of $T_{p,i}$ are obtained using a technique similar to the one presented previously (Equation 2), except that the yearly average ground load, q_y, is replaced by the average ground load up to that point, q_i. Also, it should be noted that the duration of q_i changes as the solution progresses. This value is illustrated for $i = 3$ in Figure 2 and it should not be confused with the value of $\bar{q}_{pm,i}$. Thus, Equation 2 is rewritten as follows:

$$T_{p,i} = \frac{q_i / L}{2\pi k_s} \cdot [g_{n,i} - g_{1,i}] \tag{9}$$

The g-function values in Equation 9 can be obtained from various sources. In the present work, they are obtained from the preprocessor tool developed by Cimmino and Bernier (2013) and considering a uniform temperature boundary condition at the borehole wall, as presented by Cimmino and Bernier (2014a).

Iterative Process

An iterative process is required to solve Equation 7 for two reasons. First, the time interval since the start of operation when the required bore field length is maximum is unknown a priori. Secondly, the evaluation of $T_{p,i}$ depends on the borehole height, which is also unknown at the start of the calculations. A flowchart describing the iterative process to calculate the total bore field length is shown in Figure 3.

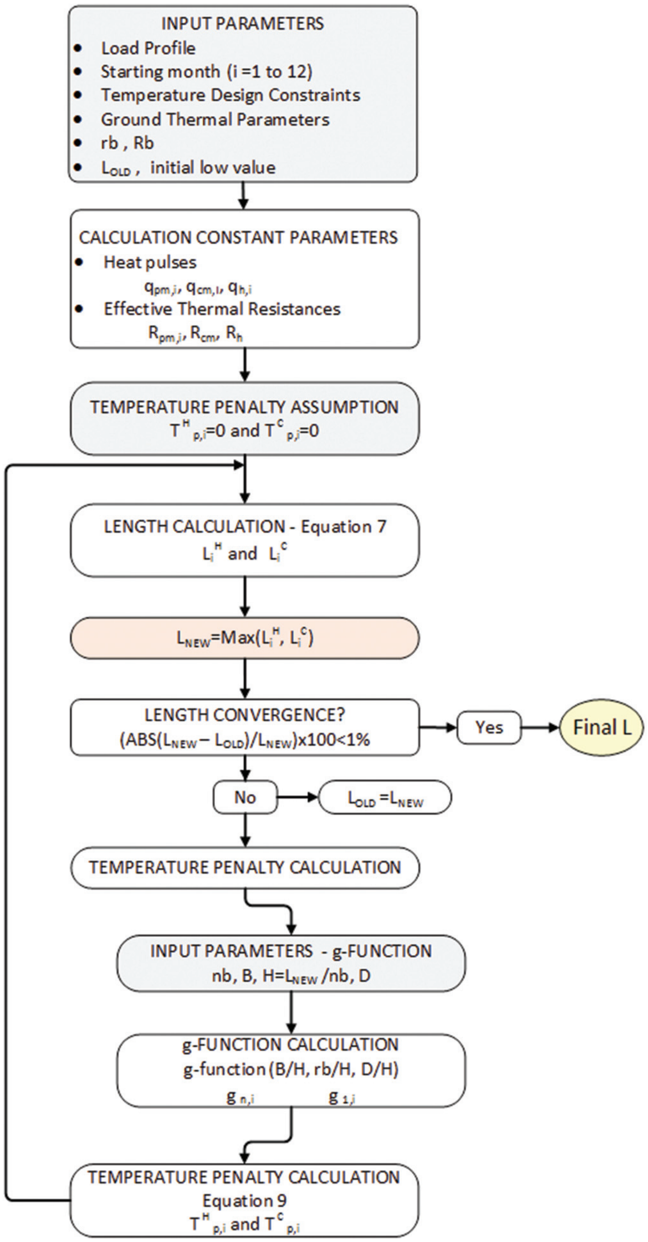

Figure 3 *Flowchart showing the iterative process to calculate* L.

First, for a given set of input parameters, ground loads are analyzed and the effective ground thermal resistance values that remain constant during the process ($R_{pm,\,i}$, R_{cm}, R_h) are calculated with the methodology described previously. In the first iteration, values of $T_{p,\,i}$ (in heating and cooling) are set to zero. Then, values of L are calculated for each month for both heating and cooling modes according to Equation 7.

The maximum value of L among the 24 values is selected as the length for the next steps (L_{NEW}). The length of individual boreholes H is then simply given by L_{NEW} / nb. The values of the g-functions $g_{n,\,i}$ and $g_{1,\,i}$ are interpolated in a precalculated table for a specific borehole layout. Then, $T^H_{p,\,i}$ and $T^C_{p,\,i}$ are calculated according to Equation 9 for the specific ratio B/H for each month. With these new values of the temperature penalties for each month, L is updated and the calculation process is repeated until convergence. The results shown in this paper converge to within 1%. Finally, it should be noted that the proposed method is adaptable to multi-year calculations.

APPLICATIONS

In this section, the proposed method is applied to determine the required length for the ground load profile shown in Figure 1. Then, using the same load profile, the effect of borehole spacing is analyzed. All calculations involved in this section use the parameters described in Table 5.

Determining *L* for Each Month Throughout the First Year of Operation

As shown in Table 6, the total length is determined using the proposed methodology for each month during the first year of operation for both heating and cooling modes (identified by letters H and C in Table 6) for a borehole spacing of 5 m (16.4 ft). In addition, two starting dates of operation are examined: January 1 and July 1. The maximum length is also evaluated for starting dates corresponding to the first day of each month for the first year of operation. These results are presented in the Appendix.

Table 5. Input Parameters

	Input Parameter	Value
Ground thermal properties	Thermal conductivity, W·m^{-1}·K^{-1} (Btu·h^{-1}·ft^{-1}·°F^{-1})	2.25 (1.30)
	Thermal diffusivity, m^2/day (ft^2/day)	0.075 (0.807)
	Undisturbed ground temperature, °C (°F)	10 (50)
Bore field	Number of boreholes	49
	Borefield pattern	7 × 7
	Borehole radius, m (in)	0.075 (2.95)
	Buried depth, m (ft)	2.5 (8.20)
	Borehole thermal resistance, m·K·W^{-1} (h·ft·°F·Btu^{-1})	0.1 (0.17)
Design constraints	Inlet heat pump temperature—heating mode, °C (°F)	0 (32)
	Inlet heat pump temperature—cooling mode, °C (°F)	35 (95)

Table 6. Equivalent Thermal Resistances, Ground Loads, and Corresponding L at Each Month for Heating (H) and Cooling (C) Modes for Two Starting Months of Operation (B = 5 m [16.4 ft])

Starting Date: January 1

i	$R_{pm,i}$, mK/W (h·ft·°F/Btu)	$\bar{q}_{pm,i}$, kW (kBtu/h)	$q_{cm,i}$, kW (kBtu/h)	$q_{h,i}$, kW (kBtu/h) H	$q_{h,i}$, kW (kBtu/h) C	H: L, m (ft)	H: T_p, K	C: L, m (ft)	C: T_p, K
1	0 (0)	0 (0)	105 (358)	239 (816)	0 (0)	**6122** (20,085)	–0.05	—	–0.07
2	0.025 (0.043)	105 (358)	92 (314)	214 (730)	0 (0)	5845 (19,177)	–0.28	—	–0.53
3	0.039 (0.067)	99 (338)	55 (188)	181 (618)	–53 (–181)	4869 (15,974)	–0.52	—	–0.96
4	0.049 (0.085)	84 (287)	0 (0)	107 (365)	–189 (–645)	2569 (8428)	–0.66	1179 (3868)	–1.14
5	0.056 (0.097)	63 (215)	–65 (–222)	80 (273)	–240 (–819)	857 (2812)	–0.53	1976 (6483)	–0.95
6	0.063 (0.109)	37 (126)	–122 (–416)	0 (0)	–328 (–1119)	—	–0.2	3072 (10,079)	–0.35
7	0.068 (0.118)	11 (38)	-151 (-515)	0 (0)	-331 (-1130)	—	0.28	**3444** (11,299)	0.47
8	0.072 (0.125)	–12 (–41)	–103 (–352)	0 (0)	–307 (–1048)	—	0.62	3100 (10,171)	1.09
9	0.077 (0.133)	–24 (–82)	–51 (–174)	57 (195)	–258 (–881)	62 (203)	0.81	2445 (8022)	1.42
10	0.080 0.138	–27 (–92)	4 (14)	112 (382)	–171 (–584)	1776 (5827)	0.82	1401 (4596)	1.41
11	0.083 (0.144)	–24 (–82)	50 (171)	151 (515)	–51 (–174)	3186 (10,453)	0.65	140 (459)	1.1
12	0.086 (0.149)	–17 (–58)	93 (317)	198 (676)	0 (0)	4836 (15,866)	0.33	—	0.55

Starting Date: July 1

i	$\bar{q}_{pm,i}$, kW (kBtu/h)	$q_{cm,i}$, kW (kBtu/h)	$q_{h,i}$, kW (kBtu/h) H	$q_{h,i}$, kW (kBtu/h) C	H: L, m (ft)	H: T_p, K	C: L, m (ft)	C: T_p, K
1	0 (0)	–149 (–509)	0 (0)	–331 (–1130)	—	0.07	**3423** (11,230)	0.11
2	–149 (–509)	–108 (–369)	0 (0)	–307 (–1048)	—	0.43	3164 (10,381)	0.68
3	–129 (–440)	–52 (–177)	57 (195)	–258 (–881)	—	0.72	2550 (8366)	1.17
4	–103 (–352)	1 (3)	112 (382)	–171 (–584)	1511 (4957)	0.86	1517 (4977)	1.36
5	–77 (–263)	48 (164)	151 515	–51 (–174)	2937 (9636)	0.78	237 (778)	1.27
6	–52 (–177)	93 (317)	198 (676)	0 (0)	4581 (15,030)	0.53	—	0.86
7	–28 (–96)	104 (355)	239 (816)	0 (0)	**5810** (19,062)	0.18	—	0.29
8	–9 (–31)	95 (324)	214 (730)	0 (0)	5446 (17,867)	–0.14	—	–0.23
9	4 (14)	55 (188)	181 (618)	–53 (–181)	4426 (14,521)	–0.35	25 (82)	–0.57
10	10 (34)	5 (17)	107 (365)	–189 (–645)	2150 (7054)	–0.36	1332 (4370)	–0.57
11	9 (31)	–62 (–212)	80 (273)	–240 (–819)	541 (1775)	–0.11	2146 (7041)	–0.18
12	3 (10)	–122 (–416)	0 (0)	–328 (–1119)	—	0.35	3271 (10,732)	0.55

The equivalent thermal resistances referred to the monthly (*) and hourly peak loads (**) are the same for every month.
(*) $R_{cm,i}$, mK/W (h-ft-°F/Btu) = 0.159 (0.275)
(**) $R_{h,i}$, mK/W (h-ft-°F/Btu) = 0.085 (0.147)

As mentioned previously, each month is considered to have 30.42 days and with 6 hours of hourly peak, which leads to constant values of $R_{cm,\,i}$ and $R_{h,\,i}$, as shown on the bottom of Table 6. However, $R_{pm,\,i}$ changes and is evaluated for each month. It varies from 0.025 to 0.086 m·K·W^{-1} (0.043 to 0.149 h·ft·°F·Btu^{-1}) over the year. The values of $\bar{q}_{pm,\,i}$, $q_{cm,\,i}$, and $q_{h,\,i}$ for both the heating and cooling modes have also been calculated for both starting dates using the load profile shown in Figure 1.

The total lengths of the heating and cooling modes and for both starting dates are given in Table 6, along with the values of T_p. Note that a positive value of T_p indicates that thermal interference is increasing the ground temperature in the bore field.

When the system starts to operate in January, the required bore field length 6122 m (20,085 ft) occurs at the end of the first month (January). For a July 1 starting date, the required bore field length (5810 m [19,062 ft]) occurs seven months later, i.e., at the end of January. This represents a 5% drop in the required bore field length. Thus, in this particular case, heat injection in the ground in the summer months is beneficial because it heats the ground, as indicated by positive values of T_p in the preceding months.

For comparison, Table 7 presents the required bore field length after 10 years of operation following the classic ASHRAE sizing approach (Equation 1). In Table 7, T_p is evaluated according to Equation 2 and is based on the annual net ground load (so that in this case q_y is equal to 7.71 kW [26.31 kBtu/h]). When sizing is based on the tenth year of operation, the required bore field length is 4921 m (16,145 ft). Thus, in this particular case, a design based on a 10-year span would have lead to an undersized bore field and operational problems during the first year of operation.

Results are also dependent on thermal diffusivity. Table 8 presents results for two other values of thermal diffusivity, 0.025 and 0.125 m^2/day (0.269 and 1.345 ft^2/day) for the tenth year of operation and a January 1 starting date. The maximum length is examined for two fixed borehole spacings and for each value of the thermal diffusivity. The borehole spacings are chosen so that the values represent the limits of negligible and strong influence for α = 0.075 m^2/day (0.807 ft^2/day) and one-year time period. For this study, the reference case has the borehole spacing for negligible influence and α = 0.075 m^2/day (0.807 ft^2/day).

For α = 0.075 m^2/day (0.807 ft^2/day), decreasing B reduces L by about 25%. The change of the maximum length is <1% when B > 10.5 m (34.4 ft), as shown in Figure 6.

For this particular study, a small value of the thermal diffusivity (α = 0.025 m^2/day) has a positive effect as it decreases L. Less heat is conducted to the surroundings, which is beneficial for the system in heating mode. The benefit is more significant when the borehole spacing is reduced. The relative change of L is 0.91 and 0.73, for B = 10.5 and 3.0 m (34.4 and 9.8 ft), respectively.

If the thermal diffusivity is increased (α = 0.125 m^2/day [1.345 ft^2/day]) and the borehole spacing is 10.5 m (34.4 ft), L increases by about 4% in comparison to the reference case, as shown in Table 8. The heat dissipation increases with the thermal diffusivity so that placing boreholes at distance B = 10.5 m (34.4 ft) leads to a negligible heat build-up in the ground. Therefore, for this particular case, to cover the heating demand, L has to be increased. However, if the borehole spac-

Table 7. Total Bore Field Length after 10 Years of Operation for Heating and Cooling Modes with Two Starting Dates—ASHRAE sizing approach (Equation 1)

Starting Date	$L_{heating}$, m (ft)	T_p, K	$L_{cooling}$, m (ft)	T_p, K
January 1	4921 (16,145)	2.13	3858 (12,657)	2.60
July 1	4894 (16,056)	2.20	3848 (12,625)	2.54

Table 8. Evaluation of the Thermal Influence in Terms of T_p and the Relative Change for Different Values of the Thermal Diffusivity (Starting Date: January 1)

α, m^2/day (ft^2/day)	0.025 (0.269)		0.075 (0.807)		0.125 (1.345)	
	Negligible Influence	Moderate Influence	Negligible Influence*	Strong Influence	Moderate Influence	Strong Influence
B, m (ft)	10.5 (34.4)	3.0 (9.8)	10.5 (34.4)	3.0 (9.8)	10.5 (34.4)	3.0 (9.8)
T_p [K]	0.24	2.71	0.63	4.36	0.88	4.78
Mode	H	H	H	C	H	C
L_{max}, m (ft)	5085 (16,683)	4096 (13,438)	5616 (18,425)	4188 (13,740)	5832 (19,134)	4533 (14,872)
Relative change [L_{max} / L_{max}(*)]	0.91	0.73	1.00	0.75	1.04	0.81

(*) Reference case: B at which the thermal influence is negligible for a period cyclic heat load of one year.

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ing is reduced to B = 3.0 m (9.8 ft), heat conduction is reduced and T_p increases significantly. Consequently, the maximum length occurs in cooling mode, and L is reduced in comparison with the values obtained from B = 10.5 m (34.4 ft) and α = 0.125 m^2/day (1.345 ft^2/day). This also has a positive change in comparison to the reference case, with a decrease of about 19%. In summary, the thermal diffusivity plays an important role in the determination of the required borehole length, among other factors such as the ground load profile and borehole spacing.

Borehole Spacing for Minimum Bore Field Length

In this section, the proposed methodology is applied to investigate the impact of borehole spacing on the required bore field length. The ground load profile presented in Figure 1 and the parameters identified in Table 5 are used in this section.

The sensitivity of the borehole spacing is investigated for two starting dates (January 1 and July 1) and two periods (during the first and tenth year). Borehole spacing is decreased from 6 to 3 m (19.7 to 9.8 ft) in intervals of 1 m (3.3 ft), which are in the range of moderate to strong influence according to the criteria defined by Hellström for a one-year periodic load (Table 3). Moreover, a borehole spacing of 3.2 m (10.5 ft) is also studied because it is the threshold identified by Hellström (1991) for a strong influence.

Results for the first and tenth year of operation are presented in Table 9, along with Figure 4 and in Table 10 in conjunction with Figure 5, respectively. For the first year of operation, L increases when B decreases and the starting date is January 1. However, an opposite behavior is observed with a July 1 starting date. As indicated in Table 9, for a July 1 starting date, the required length decreases from a value of 5853 to 5629 m (19203 to 18438 ft) when B is reduced from 6 to 3 m (19.7 to 9.8 ft) with a minimum of L = 5620 m (18,438 ft) for B = 3.2 m (10.5 ft). This result may be counterintuitive, but reduced borehole spacing increases the ground temperature in the bore field (or, in other words, there is less heat conduction to the surrounding ground), which is beneficial when the GCHP system is operating in heating mode. This is indicated by increasing values of T_p as B is decreased. However, if B is reduced further, below 3.2 m (10.5 ft), T_p becomes negative, and then the required length starts to increase. Furthermore, the required length is now obtained in February.

In the tenth year of operation, the required length L decreases with a decrease in the value of B when the system starts to operate either on January 1 or July 1, as shown in Figure 5 and Table 10. Comparing the results with different starting months, the differences of L are less than 1% for all borehole spacings. It should be noted that, for a borehole spacing of 3 m (9.8 ft), the required length during the tenth year of operation does not happen in heating mode, but in cooling mode.

The impact of reducing the borehole spacing on the value of T_p is also analyzed during the tenth year of operation for January 1 and July 1 starting dates. As the borehole spacing decreases, the absolute value of T_p increases much more than during the first year of operation, as shown in Table 10. This has an important impact on the denominator of Equation 8 and explains why higher values of L are obtained during the first year.

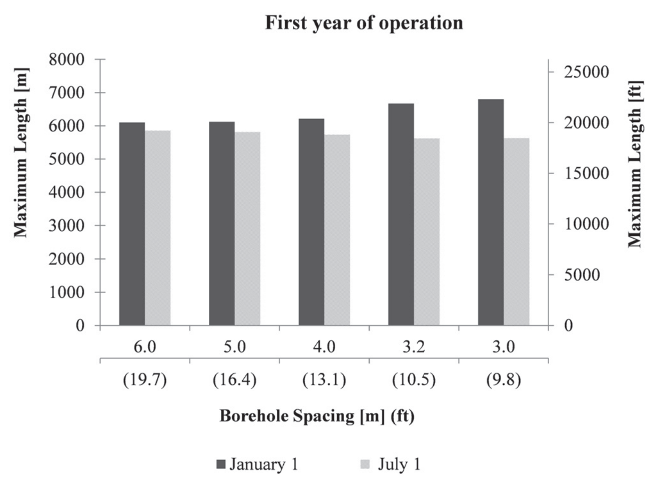

Figure 4 *Calculation of* L *for different values of* B *during the first year of operation with January 1 and July 1 starting dates.*

Table 9. Maximum Design Borehole Field Length and Temperature Penalty Values for Different Borehole Spacings for the First Year of Operation

B, m (ft)	Starting Date January 1				Starting Date July 1			
	Month	Mode	L_{max}, m (ft)	T_p, K	Month	Mode	L_{max}, m (ft)	T_p, K
6.0 (19.7)	January	H	6102 (20,020)	–0.02	January	H	5853 (19,203)	0.11
5.0 (16.4)	January	H	6122 (20,085)	–0.05	January	H	5810 (19,062)	0.18
4.0 (13.1)	January	H	6213 (20,384)	–0.19	January	H	5731 (18,803)	0.32
3.2 (10.5)	February	H	6673 (21,893)	–1.49	January	H	5620 (18,438)	0.52
3.0 (9.8)	February	H	6800 (22,310)	–1.64	February	H	5629 (18,468)	–0.46

Figure 6 shows the trend of the required bore field length for a January 1 starting date.

It can be observed that the maximum value of L occurs in heating mode when the borehole spacing is higher than the value for strong influence (B = 3.2 m [10.5 ft]). However, when the borehole spacing is lower than 3.2 m (10.5 ft), the maximum length occurs in cooling. Moreover, the maximun value of L presents similar values for the heating and cooling modes for B = 3.2 m (10.5 ft). The curve for L reaches a plateau for large values of B (>10.5 m [34.4 ft]), indicating that borehole thermal interference is negligible beyond such a spacing for 10 years of operation. Conversely, if borehole spacing is much smaller than the threshold for strong influence, then L and T_p vary significantly, as shown in Figure 6.

CONCLUSION

A methodology to calculate the total required bore field length at the end of each month for the first year of operation is presented. As shown in Equation 7, the methodology uses the same technique adopted by ASHRAE (ASHRAE 2007), with three load components with their respective durations. However, two modifications are made. First, the yearly ground

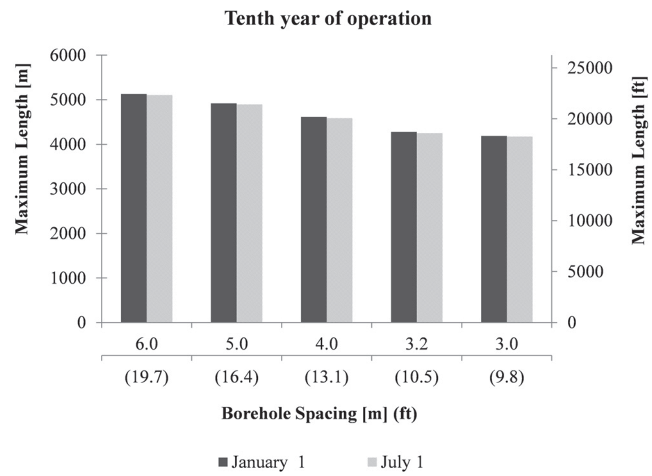

Figure 5 *Calculation of* L *for different values of* B *during the tenth year of operation with January 1 and July 1 starting dates.*

Table 10. Maximum Design Borehole Field Length and Temperature Penalty Values for Different Borehole Spacings for the Tenth Year of Operation

B, m (ft)	Starting Date January 1				Starting Date July 1			
	Month	Mode	L_{max}, m (ft)	T_p, K	Month	Mode	L_{max}, m (ft)	T_p, K
6.0 (19.7)	January	H	5128 (16,824)	1.64	January	H	5104 (16,745)	1.70
5.0 (16.4)	January	H	4921 (16,145)	2.13	January	H	4894 (16,057)	2.20
4.0 (13.1)	January	H	4616 (15,144)	2.93	January	H	4586 (15,046)	3.02
3.2 (10.5)	January	H	4280 (14,042)	3.95	January	H	4251 (13,947)	4.04
3.0 (9.8)	July	C	4188 (13,740)	4.36	July	C	4174 (13,694)	4.29

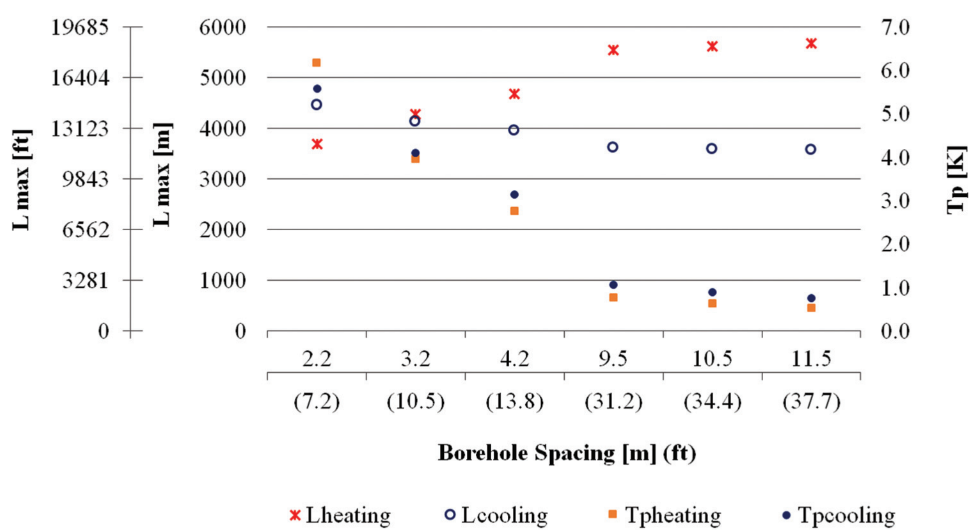

Figure 6 *Maximum* L *for extreme case (small and large borehole spacing) at the tenth years or operation. Starting date: January 1.*

load is replaced by the average ground load of the previous months. Second, the temperature penalty, T_p, is based on the average ground load of the previous months and is evaluated using g-functions. The methodology involves a three-phase process. First, ground loads need to be analyzed and properly ordered. Then, using these loads, and assuming a temperature penalty $T_p = 0$, a first set of required lengths is determined for each month. Finally, an iterative process is employed to calculate the temperature penalty in each month to obtain the final required length for the worst condition.

The proposed methodology is applied to determine the required bore field length L during the first year of operation for a particular ground load profile and prescribed input parameters.

For the particular case shown in Figure 1 with a relatively small yearly difference in heating and cooling load ($q_y = -7.71$ kW [–26.31 kBtu/h]) and the input parameters described in Table 5, the maximun length occurs during the first year of operation when the starting month is either January or July. However, starting the system in July leads to a shorter L by about 300 m (984 ft), representing a 5% reduction in the required bore field length. As shown in Appendix B, if the system is started on October 1, the required length is 16.5% higher than if the system is started on July 1. When a 10 year span is considered, the starting date of operation (January 1 or July 1) has a minimal (less than 1%) influence on L.

Results also show that it is possible to optimize the borehole spacing to have a minimum bore field length depending on the characteristics of both the load profile and the thermal properties of the ground. For the case study considered in this paper, the required bore field length can be reduced from 5853 down to 5620 m (19,203 ft to 18,438 ft) when the borehole spacing is reduced from 6 to 3 m (19.7 to 9.8 ft) and the system starts to operate in July. This result may be counterintuitive, but reduced borehole spacing increases the ground temperature in the bore field during the summer months (or, in other words, there is less heat conduction to the surrounding ground) which is beneficial when the GCHP system is operating in heating mode the next winter.

Finally, based on the findings of this work, it is recommended to change the current sizing equation in *ASHRAE Handbook—HVAC Applications* (Equation 1) to include the first year of operation as proposed in Equation 7.

APPENDIX A

Comparison with Another Software Tool

In this appendix, the proposed methodology is compared against a commercial software program on bore field sizing (Hellström and Sanner 1994). For this purpose, a known ground load profile, the thermal properties of the ground, and temperature constraints are given as input parameters (see Table 5) in the proposed methodology, as well as in the commercial software tool.

The annual ground load used for this comparison is given in Figure 1. This profile represents a cooling dominated system with a maximum hourly peak in heating and a relatively small annual ground thermal imbalance ($q_y = -7.71$ kW [–26.31 kBtu/h]).

The values of L obtained using the proposed methodology and the commercial software tool are presented in Table A.1 for different borehole spacings when the system starts to operate in January 1.

As shown in Table A.1, the difference in the calculated lengths between the proposed method and the commercial software is less than 3.5% when only the first year of operation is considered. The methodology is also tested to determine L during the tenth year of operation. The difference in the calculated lengths between the proposed method and the commercial software tool ranges from +0.7 to –8.9%. These differences are within the range of differences observed by Shonder et al. (2000) when several software tools were compared against each other.

It should be noted that the maximum length occurs in heating mode for all cases, except for the case where $B = 3.0$ m (9.8 ft) and the tenth year of operation where the maximum lengths from both methods are observed for the cooling mode. Overall, this comparison shows that the results obtained with the proposed methodology are in good agreement with an established software tool.

Table A.1 Comparison of the Proposed Method Against a Commercial Software Tool

	First Year			Tenth Year		
	L_{max}, m (ft)		Difference, %	L_{max}, m (ft)		Difference, %
B, m (ft)	Proposed Method	Commercial Software		Proposed Method	Commercial Software	
6.0 (19.7)	6102 (20,020)	5893 (19,334)	3.5	5128 (16,824)	5091 (16,703)	0.7
5.0 (16.4)	6122 (20,085)	5963 (19,564)	2.7	4921 (16,145)	4701 (15,423)	4.7
4.0 (13.1)	6213 (20,384)	6042 (19,823)	2.8	4616 (15,144)	4510 (14,797)	2.4
3.2 (10.5)	6673 (21,893)	6558 (21,516)	1.8	4280 (14,042)	4560 (14,961)	–6.1
3.0 (9.8)	6800 (22,310)	6718 (22,041)	1.2	4178 (13,707)	4585 (15,043)	–8.9

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APPENDIX B

Calculation of *L* for Different Starting Dates in the First Year of Operation

In this appendix, the maximum length is also calculated for different starting dates of operation. The first day of each month for the first year of operation have been chosen for this analysis. The results are shown in Table B.1. For all starting dates, the maximum length occurs at the end of January when *L* is evaluated for heating conditions. There is a 23% difference between the lowest and highest values of *L*.

As shown in Table B.1, the largest value of *L* occurs when the system is started on October 1. It is 16.5% higher than if the system is started on July 1 and 10.5% higher than if it is started on January 1. The operation of the system in heat extraction mode from October 1 to the end of January has decreased the ground temperature by 0.64 K, and the required GHX length (to meet the return fluid temperature constraints) is 6769 m (22,208 ft) at the end of January.

The smallest value of *L* occurs when the system is started on May 1, as shown in Table B.1. This result shows the benefits of the heat injection in the ground during the cooling months. In fact, the ground temperature increases by 0.98 K from May 1 to the end of January, which contributes to lower the required bore field length.

NOMENCLATURE

α	=	thermal diffusivity, m^2/day (ft^2/day)
B	=	borehole spacing, m (ft)
C	=	cooling mode
d	=	borehole diameter, m (in)
d_p	=	penetration depth, defined as $\sqrt{\frac{\alpha t_p}{\pi}}$, m (ft)
Fo	=	Fourier number, characteristic length d, defined as $4\alpha t / d^2$
g-function	=	dimensionless temperature response factor used to characterize the long-term response of the ground for a constant heat injection step
$g_{1,i}$	=	g-function of the single borehole at month i
$g_{n,i}$	=	g-function for a given bore field pattern at month i
G	=	G-factor, response factor from the analytical solution of an infinite cylinder in a homogeneous medium
H	=	heating mode
k_s	=	thermal conductivity, $W\cdot m^{-1}\cdot K^{-1}$ ($Btu\cdot h^{-1}\cdot ft^{-1}\cdot °F^{-1}$)
L	=	total bore field length, m (ft)
N_{adj}	=	number of adjacent boreholes
$N_{adj,CB}$	=	number of adjacent boreholes for the center borehole
$q_{cm,i}$	=	the average heat transfer to the ground at the current month, W (Btu/h)
q_h	=	the hourly peak transfer to the ground, W (Btu/h)
$q_{h,i}$	=	the hourly peak heat transfer to the ground for the current month, W (Btu/h)
q_i	=	the net average load transfer to the ground including the previous and present month, W (Btu/h)
q_m	=	the highest monthly heat transfer to the ground, W (Btu/h)
$q_{m,k}$	=	the net monthly average heat transfer to ground referred to the month k, W (Btu/h), $k = 1, \ldots, i$
$q_{pm,i}$	=	the net monthly average heat transfer to the ground referred to the previous months, W (Btu/h)
q_y	=	the net yearly average heat transfer to the ground, W (Btu/h)
r'	=	dimensionless radial distance for a hexagonal pattern, defined as $\frac{r\sqrt{2}}{d_p}$
r'_1	=	dimensionless outer radial distance r_1, defined as $\frac{r_1\sqrt{2}}{d_p}$
r'_b	=	dimensionless borehole radius r_b, defined as $\frac{r_b\sqrt{2}}{d_p}$
R_{10y}	=	the effective thermal resistance of the ground corresponding to a 10 year step, $m\cdot K\cdot W^{-1}$ ($h\cdot ft\cdot °F\cdot Btu^{-1}$)
R_{1m}	=	the effective thermal resistance of the ground corresponding to a monthly step, $m\cdot K\cdot W^{-1}$ ($h\cdot ft\cdot °F\cdot Btu^{-1}$)
R_{6h}	=	the effective thermal resistance of the ground corresponding to a 6 hour step, $m\cdot K\cdot W^{-1}$ ($h\cdot ft\cdot °F\cdot Btu^{-1}$)

Table B.1 Maximum *L* for Different Starting Dates for the First Year of Operation

Starting Date	Jan. 1	Feb. 1	Mar. 1	Apr. 1	May 1	Jun. 1	Jul. 1	Aug. 1	Sept. 1	Oct. 1	Nov. 1	Dec. 1
L_{max}, m (ft)	6122 (20,085)	5737 (18,822)	5453 (17,890)	5272 (17,297)	5249 (17,221)	5431 (17,818)	5810 (19,062)	6277 (20,594)	6618 (21,713)	6769 (22,208)	6745 (22,129)	6518 (21,385)
T_p, K	–0.05	0.35	0.73	0.98	0.98	0.69	0.18	–0.31	–0.59	–0.64	–0.56	–0.30

$R_{pm,i}$ = the effective thermal resistance of the ground corresponding to the previous month's step, m·K·W^{-1} (h·ft·°F·Btu^{-1})

$R_{cm,i}$ = the effective thermal resistance of the ground corresponding to the current monthly step m·K·W^{-1} (h·ft·°F·Btu^{-1})

t = time, s

t_p = time period of heat extraction/injection load

T_g = the undisturbed ground temperature °C (°F)

T_i = the inlet temperature to the GHX, °C (°F)

T_m = the mean fluid temperature in the borehole, °C (°F)

T_o = the outlet temperature from the GHX, °C (°F)

T_p = temperature penalty, K, a correction to account for the thermal interaction between the boreholes

Subscripts

cm = current month

h = hourly peak

i = month of operation at which *L* is evaluated, $i = 1, \ldots, 12$

max = maximum

pm = previous month

Superscripts

C = Cooling mode

H = Heating mode

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