An alternative to ASHRAE's Design Length Equation for Sizing Borehole Heat Exchangers

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ABSTRACT

An accurate determination of the required borehole length is an important step in the design of vertical ground heat exchangers used in ground-coupled heat pump systems. The ASHRAE Application Handbook presents a method to determine the design length. The method rests on three key calculations. First, effective ground thermal resistances corresponding to three thermal pulses of 10 years, 1 month and 6 hours need to be calculated. Second, the effective borehole resistance needs to be evaluated. Third, the ground temperature has to be corrected for borehole thermal interaction using a temperature penalty. The effective ground thermal resistances are evaluated using the so-called cylindrical heat source (CHS) analytical solution to transient heat transfer in the ground. This solution is relatively simple and effective ground thermal resistances can be calculated relatively easily as they do not depend on the length of the borehole. However, the CHS neglects axial heat conduction which can be a factor after a number of years. As for the temperature penalty, the handbook provides a table for estimating it. However, the table is incomplete and applies to a limited number of rectangular equally-spaced borehole configurations.

This paper proposes an alternative to the design length equation currently used in the ASHRAE handbook. In the proposed approach, the effective ground thermal resistances are evaluated using g-functions calculated based on the finite line source analytical solution. Because g-functions account for borehole thermal interaction, the correction provided by the temperature penalty is no longer needed. Furthermore, the determination of the effective ground thermal resistances is not restricted to rectangular bore fields of equally-spaced boreholes. However, as described in the paper an iterative calculation procedure is required as effective ground thermal resistances depend on the length of the borehole which is unknown a priori. In the last part of the paper, the proposed procedure is applied to determine the required length of 12×10 bore field. Results indicate that the procedure predicts a bore field length which is in the range predicted by five other design software tools.

INTRODUCTION

An accurate determination of the required borehole length is an important step in the design of vertical ground heat exchangers used in ground-coupled heat pump systems. The ASHRAE handbook (ASHRAE, 2011) proposes an equation which is based on the work of Kavanaugh and Rafferty (1997) to determine the required length of vertical ground heat exchangers. This equation has been presented by Bernier (2006) and Philippe et al. (2010) in the following form:

$$L = \frac{q_h R_b + q_y R_y + q_m R_m + q_h R_h}{T_m - (T_g + T_p)} \tag{1}$$

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where L is the required borehole length ($L = n_b \times H$, where H is the borehole length and n_b is the number of boreholes), T_m is the mean fluid temperature in the boreholes, T_g is the undisturbed ground temperature and T_p is a so-called temperature penalty. The first term in the numerator accounts for heat transfer (assumed to be in steady-state) in the borehole from the fluid to the borehole wall where R_b is the borehole thermal resistance (Kavanaugh, 2010). The next three terms in the numerator account for transient heat transfer in the ground. These three terms can be regarded as three consecutive ground thermal pulses q_y, q_m, and q_h each multiplied by their respective effective ground thermal resistance R_y, R_m, and R_h. The ground thermal pulses are considered negative when heat is extracted from the ground and positive for heat injection into the ground. The magnitude and duration of these pulses are project dependent but typically, t_1 =3650 days, t_2 =3680 days and t_f =3680.25 days. As shown in Figure 1, the ground thermal pulses correspond to three time periods. The assumed duration of the yearly average ground load, q_y , is equal to t_1. The average monthly ground load (during the month of the peak hourly load), q_m is assumed to last from t_1 to t_2. Finally, the peak hourly ground load, q_h, is assumed to last from t_2 to t_f.

ASHRAE HANDBOOK METHOD

The determination of the effective ground thermal resistances associated with the three heat pulses can be done in several ways. The method described in the ASHRAE handbook (ASHRAE, 2011) suggests using a one-dimensional analytical solution to transient heat transfer in the ground. The analytical solution used in the ASHRAE handbook is the cylindrical heat source solution (Carslaw and Jaeger, 1947). It gives the temperature distribution in the ground, including at the borehole wall, for a given heat transfer rate per unit length applied at the cylinder diameter (Bernier, 2000). Under the cylindrical heat source solution, the borehole temperature, T_w, following a heat injection rate per unit length, q_y, for a period of time t is given by:

$$T_w - T_g = q_y(G_{Fo}/k) \tag{2}$$

where G_{Fo} is the analytical solution, also known as the G-factor, Fo is the Fourier number defined as $Fo=4at/d^2$, a is the ground thermal diffusivity, d is the borehole diameter, and k is the ground thermal conductivity. Equation 2 can be written in terms of an equivalent ground thermal resistance, R_{ground}:

$$T_w - T_g = q_y R_{ground} \tag{3}$$

The effective ground thermal resistances used in Equation 1 are obtained based on the principle of superposition. For example, to determine the impact of q_y at the end of the calculation period (i.e. at t_f), q_y is first assumed to prevail from 0 to t_f. Then its thermal influence from t_1 to t_f is subtracted. A similar procedure is applied for q_m and q_h resulting in the following:

$$R_y = (G_{Fo_f} - G_{Fo_1})/k,\;\; R_m = (G_{Fo_1} - G_{Fo_2})/k\,,\;\; R_h = G_{Fo_2}/k \tag{4.a.bc}$$

with $Fo_f=4a(t_f)/d^2$, $Fo_1=4a(t_f\text{-}t_1)/d^2$, and $Fo_2=4a(t_f\text{-}t_2)/d^2$. The value of G is presented graphically in the ASHRAE Handbook. A correlation is also proposed by Bernier (2001). Finally, Philippe et al. (2010) have presented correlations to easily obtain R_y, R_m, and R_h for t_1 =3650 days, t_2 =3680 days and t_f=3680.25 days.

The determination of the design length of a bore field using Equation 1 is relatively simple. The values of R_y, R_m, and R_h are independent of the borehole length as the G-factors are based on a one-dimensional (radial) solution to ground heat transfer. Thus, contrary to the method proposed later, it does not involve an iterative solution procedure. However, as noted by Philippe et al. (2009) axial heat transfer effects start to play an important role after a few years of operation. Thus, values of R_y are not very accurate when t_1 is long. Luckily, the product $q_y R_y$ is usually small compared to the other terms in Equation 1 and any error in R_y is typically not significant in the determination of L.

The use of G-factors neglects thermal interactions among boreholes and may cause significant error in cases where there is a large annual thermal imbalance in the ground. Thus, a temperature penalty, T_p, has been introduced to account for these borehole thermal interactions. In effect, T_p is a temperature difference which corrects the value of the undisturbed ground temperature to account for the fact that boreholes do not "see" the undisturbed ground temperature when they are thermally interacting with each other. Two of the methods proposed for calculating T_p will now be described.

ASHRAE handbook method to estimate T_p

The ASHRAE handbook proposes a set of tabulated values for T_p. These values are given for a 10×10 bore field after 10 years of operation of a 350 kW (100 tons) load for various cooling and heating scenarios and borehole separations. Correction factors are provided for 1×10, 2×10, 5×5, and 20×20 bore fields. Values were obtained for a ground thermal conductivity of 1.5 W/m-K but the ground thermal diffusivity is not given.

The calculation procedure to obtain these values of T_p is defined more precisely by Kavanaugh and Rafferty (1997) and it has been analyzed by a number of authors. Fossa and Rolando (2013) have compared the temperature penalty obtained using the ASHRAE method to the ones calculated using g-functions obtained with the finite line source solution. Results indicate that the ASHRAE method underestimates the temperature penalty by 40 to 50% for some particular cases leading to a typical average underestimation of the bore field lengths of around 12%.

Kurevija et al. (2012) have also compared the length obtained by the AHRAE method with what they called the "Lund-Eskilson" model based on the finite line source analytical solution. The authors note that there are distinguishable discrepancies between the two approaches as the ASHRAE method uses a somewhat simplistic borehole interaction model. They have analyzed the effects of different borehole distances as well as different operating time for two sets of borehole arrays (21×2 and 6×7). For a bore separation of 4 m, differences of the order of 10 and 20% are noted for 21×2 and 7×6 bore fields, respectively.

Evaluation of T_p based on g-functions

Bernier et al. (2008) evaluated the temperature penalty using 3-D thermal response factors, also known as g-functions, introduced by Eskilson's (1987). A complete description of g-functions follows in the next section. Bernier et al. (2008) defined the temperature penalty as the difference between the borehole wall temperatures in the bore field and in a single borehole subjected to the same heat transfer rate per unit length. Using Eskilson's g-functions for 144 configurations, a correlation was proposed to predict the value of the temperature penalty. Although the correlation covers a large spectrum of possible configurations and operating conditions, it has some restrictions relevant to the boreholes layout which limits its application to rectangular grids and equally spaced boreholes.

Much like Bernier et al. (2008), Capozza et al. (2012) have defined the temperature penalty as the difference from the value of the thermal disturbance averaged on the borehole walls of a bore field and the value of the thermal disturbance due to a single open-field borehole. The model uses the infinite line source and a correction factor which depends on the geometrical and physical parameters. The results are compared to those obtained with the ASHRAE handbook method and the one presented by Bernier et al. (2008). Contrary to these two methods, the model presented by Capozza et al. (2012) doesn’t have any limitation on the bore field configuration. It also has wider validity range for the thermo-physical parameters compared to the method of Bernier et al. (2008). Their results are in good agreement with the ones reported by Bernier et al. (2008). Their method was applied to a case study and they showed that the ASHRAE handbook method underestimates the required length by more than 10 %.

ALTERNATIVE METHOD

As indicated earlier, Equation 1 uses effective ground thermal resistances which are based on a one-dimensional (radial) analytical solution to heat transfer from a cylinder. This assumption can lead to errors, especially for R_y. Furthermore, as shown in the previous paragraphs, the value of T_p varies significantly depending on which method is used and is most often given for rectangular equally-spaced borehole grids. An alternative method is proposed here to alleviate these deficiencies. This alternative uses g-functions to determine the design length of bore fields. When g-functions are used, thermal interference among boreholes is implicitly accounted and there is no need to apply a temperature penalty. The values of R_y, R_m, and R_h are now based on g-functions and Equations 4(a-c) take the following forms:

$$R_{gy} = \left(g_{(t_f)} - g_{(t_f - t_1)}\right)/2\pi k\,,\ R_{gm} = \left(g_{(t_f - t_1)} - g_{(t_f - t_2)}\right)/2\pi k,\ R_{gh} = g_{(t_f - t_2)}/2\pi k \qquad (5.a.b.c)$$

where g_{ti} is the g-function evaluated at $ln(t_i/t_s)$ with t_s equal to $H^2/9a$ as defined by Eskilson (1987). The subscript g has been added to the R values to indicate that they are based on g-functions. Then, with the g-function concept, the alternative design length equation is as follows:

$$L = \frac{q_h R_b + q_y R_{gy} + q_m R_{gm} + q_h R_{gh}}{T_m - T_g} \qquad (6)$$

Some design software tools use a similar g-function based approach but often with a greater number of thermal pulses (Hellström and Sanner (1994), Spitler (2000)).

The g-functions give a relation between the heat extracted from the ground per unit borehole length, q_L, and the borehole wall temperature T_w (Eskilson,1987). The borehole wall temperature is given by:

$$T_w = T_g - (q_L/2\pi k) \cdot g(t/t_s\,, r_b/H\,, B/H) \qquad (7)$$

where g represents the g-function and q_L is the heat extracted from the ground per unit borehole length. As shown in Equation 7, g-functions depend on three non-dimensional parameters: B/H, the ratio of the borehole spacing over the borehole length; r_b/H, the ratio of the borehole radius over the borehole length; and t/t_s, a non-dimensional time where t_s is a characteristic time. Typical g-functions curves are presented in Figure 2 for a 3×2 bore field as a function of $ln(t/t_s)$ for six bore field spacings, B/H, and for a particular value of r_b/H (=0.0005). The curve for $B/H=\infty$ corresponds to the g-function of a single borehole. One of the major advantages of these non-dimensional curves is that they apply to any 3×2 bore field with the same non-dimensional parameters. Eskilson (1987) provides g-function curves for a number of bore field geometries. Design software tools that use the g-function concept have a relatively large data set of g-function curves to choose from. Eskilson (1987) calculated g-functions using two-dimensional transient finite-difference equations on a radial-axial coordinate system for a single borehole in homogeneous ground. The temperature fields for each individual borehole were superimposed in space to obtain a 3-D thermal response from a borehole field for a certain configuration.

g-function curves are relatively simple to use for the determination of R_{gy}, R_{gm}, and R_{gh}. For example, the evaluation of these ground thermal resistances for a 3×2 bore field with the following characteristics: r_b = 0.05 m (2 in.), B = 5 m (16.4 ft), H = 100 m (328 ft), k =3.34 W/m-K (1.93 Btu/h-ft-°F), a = 0.096 m²/day (1.04 ft²/day) and three consecutive heat pulses of 10 years, 1 month , and 6 hours lead to the following: t_s =11574days, t_f = 3680.25 days $ln(t_f/t_s)$ = -1.14, t_1 =3650 days, $ln((t_f - t_1)/ts)$ = -5.94, t_2 = 3680 days, $ln((t_f - t_2)/ts)$ =-10.74 , g_{tf} = 12.34 , $g_{(tf-t1)}$=3.99, $g_{(tf-t2)}$ =1.55, and R_{gh} =0.074 m-K/W (0.128 h-ft-°F/Btu), R_{gm} = 0.116 m-K/W (0.201 h-ft-°F/Btu), and R_{gy} = 0.398 m-K/W (0.689 h-ft-°F/Btu).

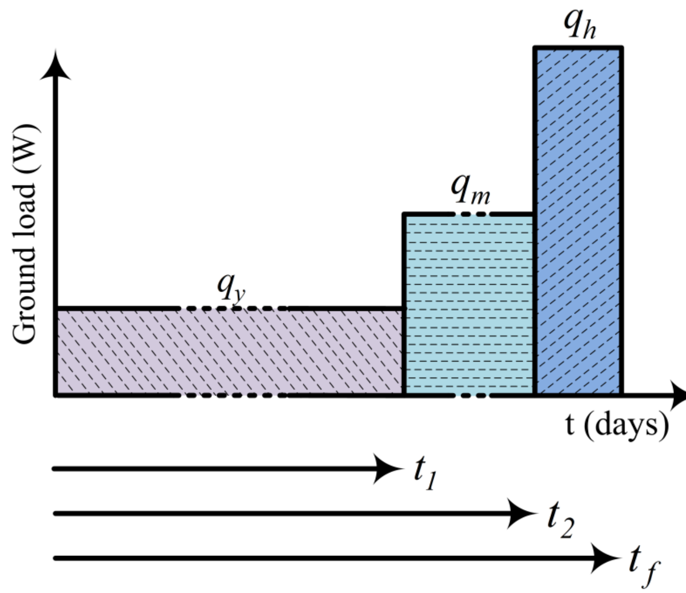

Figure 1: Three consecutive ground thermal pulses used in Equation 1.

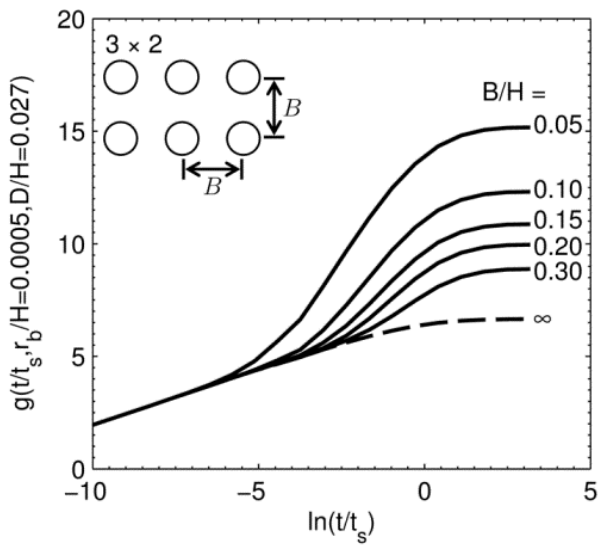

Figure 2 Six g-functions curves for a 3 × 2 bore field.

The example just presented is somewhat idealistic as H is known. In practice, in a design length calculation, H is unknown a priori and R_{gy}, R_{gm}, and R_{gh} have to be obtained iteratively. This poses some difficulties in the solution procedure. First, g-function graphs are given for specific values of r_b/H and B/H. Eskilson recommends to apply a correction factor for values of r_b/H other than the ones associated with the g-function, but it is unclear if this correction factor applies to all cases. For values of B/H other than the ones associated with the g-function, an interpolation is possible. Malayappan and Spitler (2013) used logarithmic interpolations between pre-computed g-functions for various B/H ratios. They report sizing errors of a few percent when this interpolation scheme is used. Another difficulty is the limited number of g-functions which are publicly available.

In the approach proposed here, the required g-functions are calculated "on the fly" with the proper r_b/H and B/H ratios. There are no correction factors or interpolations. The whole g-function curve does not need to be calculated since only three g-functions values are required. Furthermore, in the proposed approach, the g-functions are not restricted to rectangular equally-spaced bore fields.

g-functions are evaluated based on the methodology proposed by Cimmino and Bernier (2013, 2014) which were able to reproduce Eskilson's g-function with a high level of accuracy using the finite-line source analytical solution. A full description of this technique is out of the scope of the present paper and only key features of the method will now be described. Eskislon's g-functions are based on the assumption that all boreholes in a bore field have the same borehole wall temperature and that this temperature is uniform over the height of each borehole. In their approach, Cimmino and Bernier (2014) divided boreholes into axial segments and applied the finite line source to each of these segments. The integral mean temperature at a certain radius is obtained using a modified version of the finite line source solution proposed by Claesson and Javed (2011). A system of equation is obtained from the temporal and spatial superposition of the contribution of all borehole segments. The solution gives the wall temperature (equal for all segments of all boreholes) and the heat transfer rate of each borehole segment for a given total heat transfer rate in the bore field. In the present work, 12 axial segments are considered for each borehole. A simplified example of this procedure is provided in the Appendix.

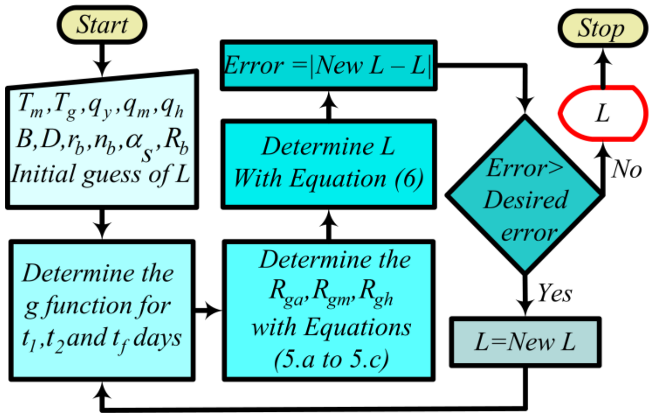

Figure 3: Flow diagram of the iterative procedure

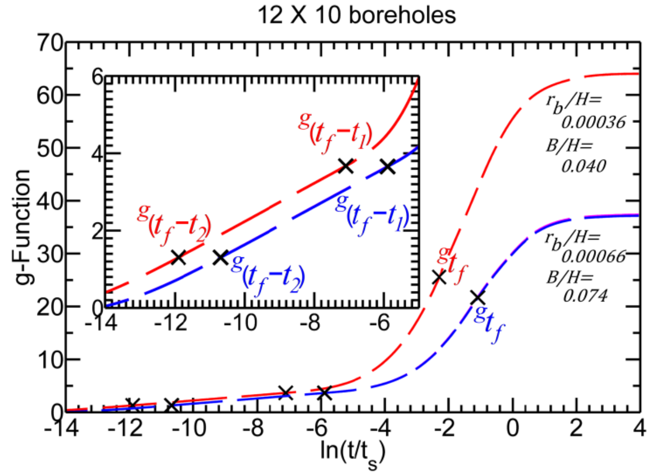

Figure 4: Determination of the three g functions related to the three thermal resistances in consecutive iterations

As mentioned earlier, the required bore length has to be determined using an iterative procedure which is presented schematically in Figure 3. This procedure can either be used for heating or cooling applications with proper signs for ground loads. The iteration procedure is comprised of five steps. In the first step, parameters are initialized and a guess value for L is chosen. Using this value of L, the three g-functions are evaluated based on $H=L/n_b$. The third step involves the determination of the three effective ground thermal resistances (Equations (5)). In the fourth step, a new length is determined using Equation (6). Finally, this new length is compared to the previous length. If the two lengths agree to within a certain tolerance, typically set at 0.01%, then calculations are stopped, if not then a new iteration is started. Typically, less than 4 iterations are required to meet the convergence criteria.

APPLICATION OF THE PROCEDURE

The proposed procedure is verified against a test case originally presented by Shonder et al. (2000) who compared five different design software tools. The case is a heating application and the bore field consists of a 12×10 grid with a borehole spacing of 6.1 m (20 ft.). The three pulses q_h, q_m, and q_y pulses are equal to -392.25 kW (-1.338e+06 BTU/h), -100.0 kW (-.341e+06 BTU/h) and -1.762 kW (-6012 BTU/h), respectively. The required borehole length is determined for t_1=3650 days, t_2=3680 days, t_f=3680.25 days. Other parameters can be found in Shonder et al. (2000) and Philippe et al. (2010).

The solution process is illustrated on Figure 4 which shows a typical g-function graph and the 9 g-function points obtained after 3 iterations. The results for the 2nd and 3rd iterations are almost identical and each cross on the bottom curve represents two superposed points. The g-function curves are not required in the solution process but are drawn on Figure 4 to illustrate the location of the point of the curve. An initial guess of L = 120×150 m (120×492.1 ft.) was chosen and the top three points correspond to the pairs $[ln((t_f\text{-}t_2)/t_s), g_{(tf\text{-}t2)}]$, $[ln((t_f\text{-}t_1)/t_s), g_{(ft\text{-}t1)}]$, $[ln(t_f/t_s), g_{tf}]$ with $t_s=H^2/9a$ = 36764 days. This leads to a new value of L = 120×82.19 m (120×269.6 ft.) and a second iteration is initiated. The process converges after 3 iterations with the final value of L = 120×81.55 m (12×267.5 ft). In their comparison exercise, Shonder et al. (2000) obtained results that ranged from 120×65 to 120×87 m (120×213.2 ft. to 120×285.4 ft.) for five different design software tools. Thus, the value obtained with the proposed procedure is in good agreement with other methodologies.

The computational time required for these three iterations is 340 s on a computer equipped with an Intel core i7 processor (2.80 GHz) and 4 GB of RAM. This relatively long computational time is due to the fact that the convergence criterion is strict (0.01%) and that 12 axial segments are used in the determination of the g-functions. Furthermore, the number of boreholes is relatively large which increases significantly the computational time associated with spatial superposition among boreholes. The same problem was solved by considering 1, 3, 6 and 9 segments for each borehole.

The resulting borehole lengths for these cases are 81.98 m (268.89 ft.), 81.86 m (268.50 ft.), 81.68 m (267.91 ft.), and 81.60 m (267.65 ft.), respectively with corresponding calculation time of 6 s, 27 s, 92 s and 195 s, respectively. Thus, in this case, computational time can be reduced by approximately 2 orders of magnitude without a significant loss in accuracy by reducing the number of borehole segments.

CONCLUSION AND RECOMMANDATIONS

This paper proposes an alternative to the design length equation currently used in the ASHRAE handbook. In the proposed approach, the effective ground thermal resistances are evaluated using g-functions calculated based on the finite line source analytical solution. Because g-functions account for borehole thermal interference, the correction provided by the temperature penalty is no longer needed. Furthermore, the determination of the effective ground thermal resistances is not restricted to rectangular bore fields of equally-spaced boreholes. However, an iterative calculation procedure is required as effective ground thermal resistances depend on the length of the borehole which is unknown a priori. New g-functions are calculated as the iterative process progresses and there is no need to apply correction factors to account for the r_b/H ratio or to interpolate between different B/H curves. The proposed procedure has been checked against a well-known test case and results indicate that the procedure predicts a bore field length which is in the range predicted by five other software tools. More work is required to determine the best compromise between the number of borehole segments and the convergence criteria to obtain a reasonable computational time with an acceptable accuracy.

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APPENDIX

In this apppendix, a simplified example is provided to better understand how g-functions are calculated in the proposed method. The example is for a bore field composed of 3 boreholes in-line (3×1), each with a radius r_b = 0.05 m (2 in.) and a height H =100 m (328 ft.). The boreholes are equally spaced by a distance B = 5 m (16.4 ft). The ground thermal conductivity, k, and diffusivity, a, are equal to 1.0 W/m-K (0.577 Btu/h-ft-°F) and 0.1 m²/day (1.07 ft²/day), respectively. The evaluation of the g-function is required for t =10 years.

The determination of the g-function is independent of T_g and q_L and values of T_g = 0 °C and q_L=$2\pi k$ are used here for convenience. In this simplified example, only one axial borehole segment is used to simplify calculations. However, in the approach proposed in this paper, 12 axial segments are used as suggested by Cimmino and Bernier (2014). Using the principle of spatial superposition, the borehole wall temperatures for boreholes 1, 2, and 3 (borehole #2 is the middle borehole) are given by:

$$T_{w1} = q_1 \times MT_{q_1,r=0.05} + q_2 \times MT_{q_2,r=5} + q_3 \times MT_{q_3,r=10} \quad \text{(A.1.a)}$$

$$T_{w2} = q_2 \times MT_{q_2,r=0.05} + q_1 \times MT_{q_1,r=5} + q_3 \times MT_{q_3,r=5} \quad \text{(A.1.b)}$$

$$T_{w3} = q_3 \times MT_{q_3,r=0.05} + q_2 \times MT_{q_2,r=5} + q_1 \times MT_{q_1,r=10} \quad \text{(A.1.c)}$$

where $MT_{q1,r=0.05}$ stands for the Mean Temperature over the segment length (= 100 m since only one segment is used in this simplified example) at a distance of 0.05 m from a line source having a heat transfer rate per unit length equal to q_1. The values of MT are obtained using the analytical solution to the finite line source proposed by Claesson and Javed (2011) with a borehole buried depth of D =4 m (13.1 ft). One of the underlying assumptions behind the g-funtion is that all borehole wall temperatures are equal. Furthermore, q_L is the average of the individual heat transfer rates:

$$T_{w1} = T_{w2}, \quad T_{w2} = T_{w3}, \; q_L = (q_1 + q_2 + q_3)/3 \quad \text{(A.2)}$$

Solving the resulting system of 6 equations and 6 unknown results in the following: q_1 = q_3 = 6.593 W/m (6.857 Btu/h-ft), q_2 = 5.664 W/m (5.891 Btu/h-ft), T_{w1}=T_{w2}=T_{w3}=8.63 °C (47.5 °F) and the corresponding g-function is:

$$g = (T_w - T_g) \times (2\pi k/q_L) = 8.63$$